

武汉理工大学考试试卷 (A 卷)

2018~2019 学年 1 学期 概率论与数理统计 B 课程 闭卷

时间 120 分钟, 48 学时, 3.0 学分, 总分 100 分, 占总评成绩 70% 2019 年 1 月 4 日

题号	一	二	三	四	五	六	七	八	九	十	合计
满分	15	15	10	10	10	10	10	10	10		100
得分	15	15	10	10	10	10	10	9	10		99

$\Phi(0) = 0.5$, $\Phi(1.645) = 0.95$, $\chi_{0.025}^2(15) = 27.488$, $\chi_{0.975}^2(15) = 6.262$,
 $\chi_{0.025}^2(16) = 28.845$, $\chi_{0.975}^2(16) = 6.908$, $t_{0.025}(7) = 2.3646$, $t_{0.025}(6) = 2.4469$

得分 15

I. Fill in the blanks. (5×3')

- Suppose that $P(A) = 0.5$, $P(B) = 0.6$, $P(B|A) = 0.8$. Then $P(A \cup B) = 0.7$.
- Suppose that $X \sim N(2, \sigma^2)$, and $P\{2 < X \leq 4\} = 0.3$, then $P\{X < 0\} = 0.2$.
- Suppose that $X \sim P(\lambda)$, $\lambda > 0$, and $E[(X-2)(X-3)] = 2$, then $\lambda = 2$.
- Suppose that $X_1, X_2, \dots$ are a random sample from standard normal distribution. Let $Y = (X_1 + X_2 + X_3)^2 + (X_4 + X_5 + X_6)^2$, then determine the value of $C = \frac{1}{2}$ such that CY has a χ^2 distribution.
- Suppose that a random sample of 16 observations is taken from a normal distribution for which both the mean μ and the variance σ^2 are unknown. We got $\bar{x} = 0.081$ and $s = 6.2022$, then the 95% confidence interval for the deviation σ is $(4.5816, 9.5992)$.

得分 15

II. Please choose only ONE letter among A, B, C, D, and put it in the bracket within that question. (5×3')

- Let A, B be events, then we have that $P(A - B) =$
~~A. $P(A) - P(B)$~~ B. $P(A) - P(B) + P(\bar{A}B)$
~~C. $P(A) + P(AB)$~~ D. $P(A) + P(\bar{B}) - P(AB)$
- Given that $P(A) = 0.3$, $P(B) = 0.5$, $P(AB) = 0.15$, then
A. $P(B|A) = P(AB)$ B. $P(B|\bar{A}) = P(BA)$
C. $P(A|B) = P(A)$ D. $P(A|\bar{B}) = P(\bar{B})$

$$P(AB) = 0.15 = 0.3 \times 0.5$$

$\rho = 0$

- If random variables X and Y has correlation coefficient $\rho = 0$, then (C)
~~A. X and Y are independent~~ B. X and Y are not necessarily correlated
C. X and Y are definitely uncorrelated D. X and Y are correlated

- Suppose that $X_1, X_2, \dots$ are a random sample from the population $X \sim N(\mu, \sigma^2)$, which one is the unbiased estimator of σ^2 (B)

A. $\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2$

B. $\frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$ $X \sim N(\mu, \sigma^2)$

C. $\frac{1}{n} \sum_{i=1}^n X_i^2$

D. $\bar{X}^2$

- Let X be a random variable and $Var(X) = \sigma^2$, then $P[|X - E(X)| < 3\sigma]$ (A)

A. $\geq \frac{8}{9}$

B. $< \frac{8}{9}$

C. $> \frac{1}{9}$

D. $\leq \frac{1}{9}$

$\geq 1 - \frac{\sigma^2}{9\sigma^2} \geq \frac{8}{9}$

得分 10

III. (10') A component of computer is provided by three factories A, B, and C.

Suppose that 45 percent of the components are provided by factory A, 36 percent by factor B and 19 percent by factory C. Suppose further that the defective rate of the components provided by factory A, factory B and factory C is 0.05, 0.04 and 0.02 respectively.

- Find the probability that a randomly selected one component is defective.
- Suppose that one component is selected at random and it is found to be defective. Determine the probability that it is provided by factory C.

解 (1) 假设抽到产品为有质量问题的事件为 X

$$\begin{aligned} P(X) &= P(X|A) \cdot P(A) + P(X|B) \cdot P(B) + P(X|C) \cdot P(C) \\ &= 0.45 \times 0.05 + 0.36 \times 0.04 + 0.19 \times 0.02 \\ &= 0.0407 \end{aligned}$$

$$\begin{aligned} P(C|X) &= \frac{P(X|C) \cdot P(C)}{P(X)} = \frac{0.19 \times 0.02}{0.0407} \\ &= 0.093 \end{aligned}$$

得分

10

IV. (10') Suppose that the p.d.f. of a random variable X is as follows:

$$f(x) = Ce^{-\frac{|x|}{a}}, (a > 0)$$

(1) Find the value of the constant C ; (2) Find the d.f. $F(x)$ of X ; (3) Find the value of $P(|X| < 2)$

解 (1) $\because$ p.d.f. $f(x) = ce^{-\frac{|x|}{a}}, (x \in \mathbb{R}, a > 0)$

$$\text{且 } \int_{-\infty}^{+\infty} f(x) dx = 2 \int_0^{+\infty} f(x) dx = 2 \int_0^{+\infty} ce^{-\frac{x}{a}} dx$$

$$= 2ac$$

$$\because 2ac = 1 \Rightarrow c = \frac{1}{2a}$$

$$(2) F(x) = \int_{-\infty}^x f(t) dt$$

$$1^\circ \text{ 当 } x < 0 \text{ 时 } F(x) = \frac{1}{2a} \int_{-\infty}^x e^{\frac{t}{a}} dt = \frac{1}{2} e^{\frac{x}{a}}$$

$$2^\circ \text{ 当 } x \geq 0 \text{ 时 } F(x) = \frac{1}{2} + \frac{1}{2a} \int_0^x e^{-\frac{t}{a}} dt = 1 - \frac{1}{2} e^{-\frac{x}{a}}$$

$$\therefore F(x) = \begin{cases} \frac{1}{2} e^{\frac{x}{a}} & x < 0 \\ 1 - \frac{1}{2} e^{-\frac{x}{a}} & x \geq 0 \end{cases}$$

$$P(|X| < 2)$$

$$= P(-2 < X < 2)$$

$$= F(2) - F(-2) + P(X=2)$$

$$= 1 - \frac{1}{2} e^{-\frac{2}{a}} - \frac{1}{2} e^{-\frac{2}{a}} + 0$$

$$= 1 - e^{-\frac{2}{a}}$$

得分

10

V. (10') Suppose that X has an exponential distribution with parameter 2, which

denoted by $X \sim E(2)$. Determine the p.d.f. of $Y = 1 - e^{-2X}$.

解: $\because X \sim E(2) \therefore$ p.d.f. of X is $f(x) = \begin{cases} 2e^{-2x} & x > 0 \\ 0 & x \leq 0 \end{cases}$

$$\text{当 } x > 0 \quad y = 1 - e^{-2x} \in (0, 1)$$

单调的 $x = h(y) = -\frac{1}{2} \ln(1-y)$ $|x'_y| = |h'(y)| = \frac{1}{1-y}$ $y = 1 - e^{-2x}$ 在 $x \in (0, +\infty)$ 上是绝对

由式知

$$f_Y(y) = \begin{cases} f[h(y)] |h'(y)| \\ 0 \end{cases}$$

$$0 < y < 1 \Leftrightarrow \begin{cases} 1 \\ 0 \end{cases} \quad \begin{matrix} 0 < y < 1 \\ \text{其它} \end{matrix}$$

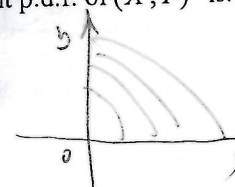
$$\therefore Y \sim U(0, 1)$$

得分

10

VI. (10') Suppose that the joint p.d.f. of (X, Y) is:

$$f(x, y) = \begin{cases} \frac{1}{2}(x+y)e^{-(x+y)}, & x > 0, y > 0, \\ 0, & \text{otherwise} \end{cases}$$



1) Determine whether or not X and Y are independent and give your reason.

2) Determine the p.d.f. of $Z = X + Y$.

$$\text{解 (1)} \because f_{X,Y} = \begin{cases} \frac{1}{2}(x+y)e^{-(x+y)}, & x > 0, y > 0 \\ 0 & \text{其它} \end{cases}$$

$$1^\circ \text{ 当 } x \leq 0 \text{ 时 } f_X(x) = 0$$

$$2^\circ \text{ 当 } x > 0 \text{ 时 } f_X(x) = \int_0^{+\infty} f_{X,Y}(x,y) dy = \frac{1}{2} e^{-x}(x+1)$$

$$\therefore f_X(x) = \begin{cases} \frac{1}{2} e^{-x}(x+1) & x > 0 \\ 0 & x \leq 0 \end{cases}$$

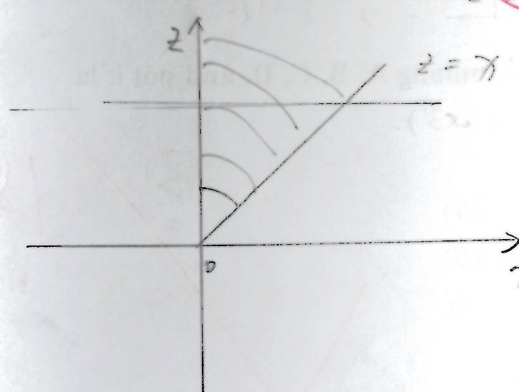
$$\text{同理 } f_Y(y) = \begin{cases} \frac{1}{2} e^{-y}(y+1) & y > 0 \\ 0 & y \leq 0 \end{cases}$$

$$\therefore f_X(x) \cdot f_Y(y) = \frac{1}{4} e^{-(x+y)}(x+1)(y+1) \neq \frac{1}{2}(x+y)e^{-(x+y)}$$

$\therefore X$ 和 Y 不是独立的

$$(2) Z = X + Y \quad \therefore Y = Z - X \quad \text{由 } \begin{cases} x > 0 \\ y > 0 \end{cases} \Rightarrow \begin{cases} x > 0 \\ z-x > 0 \end{cases} \Rightarrow \begin{cases} x > 0 \\ z > x > 0 \end{cases}$$

$$\Rightarrow \begin{cases} x > 0 \\ z-x > 0 \end{cases} \Rightarrow \begin{cases} x > 0 \\ z > x > 0 \end{cases}$$



$$f_Z(z) = \int_0^z f_X(x, z-x) dx$$

$$= \int_0^z \frac{1}{2} z e^{-z} dx$$

$$= \frac{1}{2} z^2 e^{-z} (z > 0)$$

$$\therefore f_Z(z) = \begin{cases} \frac{1}{2} z^2 e^{-z} & z > 0 \\ 0 & z \leq 0 \end{cases}$$

余崇东 qj1703 19

VII. (10') Suppose that a machine produces a nondefective item with probability 0.8. What is the smallest number of items that have to be produced in order for the probability to be at least 0.9 that the proportion of nondefective in this batch of items will be between 76% and 84%? (Please use the central limit theorem to give the estimation)

解: 设 X_i 表示第 i 次抽取时获得的无缺陷品个数 ($i=1, \dots, n$)

设一共抽取 n 次 由题 $X_i \sim B(1, 0.8)$ 由题

$$\therefore E(X) = 0.8 \quad D(X) = 0.8 \times 0.2 = 0.16$$

由中心极限定理知:

$$\frac{1}{n} \sum_{i=1}^n X_i \sim N(\mu, \frac{\sigma^2}{n}) \text{ 即 } N(0.8, \frac{0.16}{n})$$

$$\therefore \frac{\frac{1}{n} \sum_{i=1}^n X_i - 0.8}{0.4 / \sqrt{n}} \sim N(0, 1)$$

$$\therefore \Phi(0.1\sqrt{n}) \geq 0.95$$

$$\text{又 } \Phi(1.645) = 0.95$$

$$\therefore 0.1\sqrt{n} \geq 1.645 \Rightarrow n \geq 270.6025$$

$$\therefore n_{\min} = 271$$

VIII. (10') Suppose that $X_1, X_2, \dots, X_n$ are a random sample from population X for which the p.d.f. is

$$f(x; \theta) = \begin{cases} \frac{\theta^2}{x^3} e^{-\frac{\theta}{x}}, & \text{for } x > 0, \\ 0, & \text{otherwise,} \end{cases}$$

where parameter $\theta > 0$ is unknown and that $x_1, x_2, \dots, x_n$ are observed values of $X_1, X_2, \dots, X_n$.

(1) Determine the moment estimators of θ .

(2) Determine the maximum likelihood estimator of θ .

$$\text{解 (1)} \quad E(X) = \int_0^{+\infty} x f(x, \theta) dx = \int_0^{+\infty} \frac{\theta^2}{x^2} e^{-\frac{\theta}{x}} dx = \theta$$

若用矩估计方法估计

$$\text{由 } E(X, \theta) = \theta = A = \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i, \quad \hat{\theta} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$(2) \text{ 设 } l(\theta) = \ln f(x_1, x_2, \dots, x_n, \theta) = 2n \ln \theta - 3 \sum_{i=1}^n \ln x_i - \frac{\theta}{x_i}$$

$$\therefore l'(\theta) = \frac{2n}{\theta} - \left(\sum_{i=1}^n \frac{1}{x_i} \right), \text{ 若用最大似然法估计有}$$

$$l'(\theta) = 0 \Rightarrow \hat{\theta} = \frac{2n}{\sum_{i=1}^n \frac{1}{x_i}} = \frac{2}{\bar{x}}$$

IX. (10') Suppose that lifetime, in hours, of an electric component have a normal distribution. Test the hypothesis that the average lifetime of an electric component is 23.8 hours if the lifetimes of a random sample of 7 components are 26.7, 22.0, 24.1, 21.0, 27.2, 25.0 and 23.4 hours. Use a 0.05 level of significance to confirm that the distribution of the lifetime is normal. $\alpha = 0.05$

解: 原假设: $H_0: \mu = \mu_0$ (normal)

备择假设 $H_1: \mu \neq \mu_0$ (innormal)

由于 σ 是未知的, 故选择统计量:

$$\frac{\bar{x} - \mu_0}{s / \sqrt{n}} \sim t(n-1)$$

由数据可计算出 $\bar{x} = 24.2$

$$(2.5, 2.2, 0.1, 3.2, 3, 0.8, 0.8) \quad S = \sqrt{\frac{1}{6} \sum_{i=1}^7 (x_i - \bar{x})^2} = 2.2956$$

$$\therefore \frac{\bar{x} - \mu_0}{s / \sqrt{n}} = \frac{24.2 - 23.8}{2.2956 / \sqrt{7}} = 0.461$$

$$\text{又 } t_{0.025}(6) = 2.4469 > 0.461$$

可以认为寿命是处于正常值。

$$\frac{\bar{x} - \mu}{s / \sqrt{n}} \sim t(n-1)$$